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MULTI-TRIANGULAR SAMPLING PLANS FOR PARTIAL DIALLEL CROSSES

C. SUBBA RAO[1] and PREM NARAIN[2]

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SUMMARY

Partial diallel crosses based on multi-triangular design (MTD) have been constructed and analysed when the number of parental lines is of the form $p(p-1)(p-2)(p-3)/24$ with 'p' as an integer greater than 4. Designs based on multi-triangular association plans are more efficient than circulent designs of Kempthorne and Curnow.

Keywords : Partial Diallel Crosses; Multi-Triangular Design; Circular Design; Least Square Technique.

1. Introduction

The partial diallel crossing system was initially dealt by Kempthorne and Curnow [3] Fyfe and Gilbert [2] and Curnow [1]. Fyfe and Gilbert [2] constructed such crosses with the help of 'triangular' designs in which the number of lines could be of the form $p(p-1)/2$ where 'p' is an integer. Narain *et al.* [4] gave the procedures of constructing and analysing partial diallel crosses based on extended triangular design, where the number of parental lines is of the form $p(p-1)(p-2)/6$ with 'p' as an integer greater than 3. In the present investigation, partial diallel crosses based on multi-triangular design (MTD) have been constructed and analysed when the number of parental lines is of the form $p(p-1)(p-2)(p-3)/24$ with 'p' as an integer greater than 4.

1. Scientist S-1 (Statistics), All India Coordinated Rice Improvement Project, Rajendranagar, Hyderabad-500 030, A.P.
2. Director, IASRI, New Delhi-110 012.

2. Construction

Sampling plans for partial diallel crosses can be obtained in a manner similar to that of triangular designs of Fyfe and Gilbert (1963) and of the extended T design, of Narain *et al.* [4].

2.1 *Muiti-triangular Sampling Plans for Partial Diallel Crosses*

Suppose the number of parents 'n' be of the form p_{c_4} or $[p\ (p-1)\ (p-2)\ (p-3)]/24$ where 'p' is an integer. Denote a parent by quadruplet $abcd$ where 'a' takes any value from 4 to p, 'b' takes from 3 to $(a-1)$, 'c' takes from 2 to $(b-1)$ and 'd' takes from 1 to $(c-1)$. All the parents can be numbered of into $(p-3)\ (p-2)/2$ different triangles of different orders from 1 to $(p-3)$. Number of parents in each triangle is given by $(p-1-j)c_2$ and the order of the triangle is given by $(p-2-j)$, where $j = 1, 2, \ldots, (p-3)$. The number of parents

$$n = \sum_{j-1}^{p-3} j \times (p-1-j)c_2 = \frac{p(p-1)\ (p-2\ (p-3)}{24}$$

Illustration : Consider $n = 126$ and $p = 9$ so that $n = (9 \times 8 \times 7 \times 6)/24$. The triangles are obtained by decreasing the digits in order from right side onwards (Table 1).

TABLE 1

Type of triangle	*Quadruplets*						*No. of quadruplets*
T_1	9876	9875	9874	9873	9872	9871	
		9865	9864	9863	9862	9861	
			9854	9853	9852	9851	21
				9843	9842	9841	
					9832	9831	
						9821	
T_2		9765	9764	9763	9762	9761	
			9754	9753	9752	9751	15
				9743	9742	9741	
					9732	9731	
						9721	
T_3			9654	9653	9652	9651	
				9643	9642	9641	10
					9632	9631	
						9621	
T_4				9543	9542	9541	

Table 1 (*contd. on page* 207)

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				9532	9531	6
					9521	
T_5				9432	9431	
					9421	3
T_6					9321	1
T_2	8765	8764	8763	8662	8761	
		8754	8753	8752	8751	
			8743	8742	8741	15
				8732	8731	
					8721	
T_3		8654	8653	8652	8651	
			8643	8642	8641	10
				8632	8631	
					8621	
T_4			8543	8542	8541	
				8532	8531	6
					8521	
T_5				8432	8431	3
					8421	
T_6					8321	1
T_3		7654	7653	7652	7651	
			7643	7642	7641	10
				7632	7631	
					7621	
T_4		7543	7542	7541		6
			7532	7531		
				7521		
T_5				7432	7431	3
					7421	
T_6					7321	1
T_4			6543	6542	6541	
				6532	6531	6
					6521	
T_5				6432	6431	3
					6421	
T_6					6321	1
T_5				5432	5431	3
					5421	
T_6					5321	1
T_6					4321	1

We obtain the triangle of different orders as shown in Table 2.

TABLE 2—NUMBERING OF THE PARENTS INTO TRIANGLES FOR CONSTRUCTION OF MTD WITH $n = 126, p = 9$

Type of triangle (jth)	*Order of triangle*	*No. of quadruplets within triangle*	*No. of triangles*	*No. of parents*
T_1	6 × 6	21	1	21
T_2	5 × 5	15	2	30
T_3	4 × 4	10	3	30
T_4	3 × 3	6	4	24
T_5	2 × 2	3	5	15
T_6	1 × 1	1	6	6
				126

Imposing conditions on *a, b, c* and *d* of the quadruplets, we get four different designs.

Design I. We sample all the crosses of type *abcd* × *efgh*, where *a, b, c, d, e, f, g, h* are all distinct. The number of times each parents is involved in crossing with other parents is given by

$$s_1 = {}^{(p-4)}C_4$$

The resulting sample would consist of $ns_1/2$ crosses.

Design II. We sample all the crosses of type *abcd* × *efgh*, where one of the letters (*a, b ,c, d ,e, f, g* and *h*) is common. The number of times each parents is involved in crossing with other parents, denoted by s_2 is given by $s_2 = 4\ {}^{(p-4)}C_3$ and the resulting sample would consist of $ns_2/2$ crosses.

Design III. In this design, we sample all the crosses of type *abcd* × *efgh*, where two of the letters are common. The number of times each parent is involved in crossing with other parents, s_3, is given by $s_3 = 6 \times {}^{(p-4)}C_2$ and the resulting sample would consist of $ns_3/2$ crosses.

Design IV. In this design, we sample all the crosses of type *abcd* × *efgh*, where three of the letters are common $s_4 = 4\ (p - 4)$, sample = $ns_4/2$ crosses.

It can be shown that given the value of n, the number of parental lines, there can be only one value of 'p' satisfying

$$n = \frac{p(p-1)(p-2)(p-3)}{24}$$

This also fixes, then the value of 's' the number of times each line is involved in crosses with other lines. Table 3 illustrates some examples of n and p along with the values of s for the four designs constructed above.

TABLE 3—NUMBER OF PARENTS AND NUMBER OF TIMES EACH PARENT INVOLVES IN CROSSING IN FOUR DESIGNS

No. of parents	*p*	*s*			
n		*Design I*	*Design II*	*Design III*	*Design IV*
5	5	—	—	—	4
15	6	—	—	6	8
35	7	—	4	18	12
70	8	1	16	36	16
126	9	5	40	60	20
210	10	15	80	90	24
330	11	35	140	126	28

3. Analysis

To analyse the partial diallel crosses constructed above the least square technique is used. We assume that only one set of F_1 crosses is considered. The mean yield of the cross between ith $\times$ jth parent is expressed as

$$\overline{Y}_{ij} = \mu + g_i + g_j + s_{ij} + e_{ij},$$

where μ is the effect due to overall mean, g_i and g_j are the *g.c.a.* effects due to ith and jth parents respectively, s_{ij} is the *s.c.a.* effect due to the cross $(i \times j)$ and e_{ij} is the random error. We assume that $\sum_{i=1}^{n} g_i = 0$, $\sum_{j=1}^{n} s_{ij} = 0$ for each i and that g_i, s_{ij} are independently normally distributed with zero means and variances 6_g^2, 6_s^2, 6_e^2 respectively. Let the observations and parameters be alternatively represented by the relation

$$Y = X\beta + e$$

where Y = a vector of observed yields of $(i \times j)$, X = the design matrix,

β = a vector of parameters. The least square estimates of the parameters are given by

$$\beta = (X'X)^{-1} X^1 Y$$

A detailed description of procedures of the analysis is given by Subba Rao [5]. It is apparent from the description that the effect of a particular design of partial diallel crosses on the estimation of *g.c.a.* effect and its S.E. would depend upon the elements of the inverse matrix A^{-1}. The elements of A and A^{-1} for $n = 35$ (with $p = 7$) and $n = 15$ (with $p = 6$) are presented in Table 4.

TABLE 4—THE ELEMENTS OF A AND A^{-1} FOR $n = 35$ AND $n = 15$ FOR FOUR DESIGNS

Elements	*Design I* $n = 35$	*Design II* $n = 35$		*Design III* $n = 15$		*Design III* $n = 35$		*Design IV* $n = 15$		*Design IV* $n = 35$	
Diagonal	——	4	0.3714	6	0.1968	18	0.571	8	0.1335	12	0.0979
abcd × *efgh*	——	—	—	—	—	—	—	—	—	—	—
abcd × *afgh*	——	1	—0.1286	—	—	0	0.0000	—	—	0	—0.0020
abcd × *abgh*	——	0	—0.0175	1	—0.0413	1	—0.0032	0	0.0003	0	0.0002
abcd × *abch*	——	0	0.0381	0	0.0064	0	0.0000	1	—0.0165	1	—0.0069

The average variances of different designs for $n = 15$ and 35 were determined and are presented in Table 5.

TABLE 5—AVERAGE VARIANCES OF DIFFERENT DESIGNS

Type of design	*n*	*s*	*Av. Var.*	*S*	*Av. Var.*	*S*	*Av. Var.*
	15	6		8			
MTD			0.2109		0.1476		
K and C			0.2407		0.1545		
TD			0.2109		0.1500		
Factorial			0.2000		0.1468		
	35	4		18		12	
E.T.D			0.7825		0.116		0.185
MTD			0.7216		0.1174		0.1804
K and C			2.1006		0.1214		0.2102

MTD : Multi-triangular design, K and C : Kempthorne and Curnow, TD : Triangular design.

unusually high — funny

For $n = 15$ and different values of s, designs based on multi-triangular association plans are more efficient than circulent designs (C.D.) of Kempthorne and Curnow [3] and equally efficient that of Fyfe and Gilbert [2] and less efficient to factorial designs. For $n = 35$ and different values of s, multi-triangular designs are more efficient than C.D. of Kempthorne and Curnow [3] and equally efficient with that of ETD developed by Narain *et al.* [4]. Thus these sampling plans could be used as alternatives to the existing ones.

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